Validation and application of sand pile modeling of multiseeded HTS bulk superconductors

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Abstract—Sand pile and Bean models have already been applied to describe single grain HTS bulks. An extension to that approach was used to model multiseeded bulks, needed for several practical applications as electric motors or flywheels with superconducting bearings. The use of genetic algorithms was then proposed in order to determine intra- and intergrain current densities, and application to two and three seeds samples using trapped flux experimental measurements was exemplified. However, this model assumed some simplifications, as equal properties in grain boundaries between neighboring grains. In this paper an extension to this methodology is proposed and evaluated by analyzing measurements performed in plans at different distances from surfaces of samples with three seeds. Discussion of its influence on a practical application is also explored.

Index Terms— Genetic algorithms, multiseeded superconductors, sand pile model, trapped flux.

I. INTRODUCTION

The advent of motors or other power devices employing trapped flux superconducting bulk materials relies on the ability to model these materials properly in the design stage [1]-[3]. This is particularly relevant in ironless devices, as magnetic flux lines are spread in surrounding medium. In a previous work, the force developed by an all superconducting linear motor was derived by averaging magnetic flux density of two trapped flux magnets over the volume of superconducting coils in the armature of the motor [4]. This flux density was calculated by means of sand pile model [5], where persistent currents are assumed to flow in concentric loops parallel to sample edges. Biot-Savart law is then used for straightforward calculation of flux density components in any point in space. Originally, only single grain materials were considered, and an extension of these models to multiseeded materials was proposed in [6], by combining intra- and intergrain current densities, as shown in Fig. 1. Determination of current densities from measurements of trapped flux was then carried out by genetic algorithms (GA), a class of computational models inspired in natural evolution that perform parallel search [7]. It was assumed that grain boundaries shared the same properties, thus a single intergrain current was considered. This limitation is now addressed in the present work by considering loops between adjacent grains, for a three seeds sample.

The paper is organized as follows: methods are reviewed in the next section and the proposed extension of sand pile modeling is also presented. In section III, the new methodology is evaluated and discussed by analysis of experimental measurements of trapped flux at different heights. Conclusions are drawn in the last section of the paper.

II. METHODS

A. Sand pile modeling of one single grain

Each grain (conceptually consisting on a set of intra- or intergrain current loops that share the same current density, as already represented in Fig. 1) generates a field \( \mathbf{B} \) at any point \( \mathbf{P} \) in space which can be derived by Biot-Savart law. Simple expressions for \( \mathbf{B} \) components are derived in [5] or [7]. Bean model [8] is considered, meaning that current flowing in each loop is determined by constant current density of the sample, \( J_C \).

B. Modeling of three seeds samples

Using the concept represented in Fig. 1, the field of a three seeds sample at a distance \( Z_0 \) from surface corresponds to the superposition of four fields [3], each originated by its critical current density, \( J_{C1} \) to \( J_{C4} \), as exemplified in Fig. 2.

Fig. 1. Sand pile modeling of multiseeded samples consisting on the superposition of intragrain (top) and intergrain (bottom) current loops. Illustration for a three seeds sample.
trapped flux shown in Fig. 3.a) (sample #1), current densities \( \mathbf{J} = [88.20, 103.90, 87.51, 81.07] \text{ A/mm}^2 \) were found as the best solution (see details in [3]), with \( \varepsilon = 18.8\% \). The modeled sample is represented in Fig. 3.b).

This approach is adequate for practical applications in which samples show fairly good balance between peaks and/or valleys in flux density profiles. This is not the case, however, of sample #2 in Fig. 4.a), where valleys are clearly distinct. In this case, the best solution, \( \mathbf{J} = [83.48, 59.21, 69.55, 46.98] \text{ A/mm}^2 \), leads to \( \varepsilon = 19.8\% \), and modeling results are shown in Fig. 4.b). The distinction between the valleys is lost.

C. Genetic algorithms

The purpose of using GA is to determine current densities of a sample, \( \mathbf{J} = J_{c1}[J_{c2}]J_{c3}[J_{c4}] \), from real measurements of normal component of trapped flux in a plan located at \( z = Z_0 \), \( B_z(J', x, y, Z_0) \). Each individual, corresponding to a possible solution of this problem, is built by the concatenation of intragrain current densities followed by intergrain current density, i.e. \( \mathbf{J} = J_{c1}[J_{c2}]J_{c3}[J_{c4}] \). Flux density corresponding to \( \mathbf{J} \) is defined as \( \tilde{B}_z(J, x, y, Z_0) \). GA applies a fitness function, \( \varepsilon(J) \), to evaluate each solution and in this work the normalized root mean square deviation (NRMSD) between these distributions over the considered domain is used,

\[
\varepsilon(J) = 100\% \sqrt{\frac{\left( \frac{1}{N} \sum_{i=1}^{N} (B_i - \tilde{B}_i(J, x, y))^2 \right)}{(B_{\text{max}} - B_{\text{min}})}}
\]  

where \( N \) is the number of points evaluated and \( B_{\text{max}} \) and \( B_{\text{min}} \) are the maximum and minimum values of \( B_i \) in the domain, respectively. The only considered restriction is \( J_{c_m} - J_{c_4} > 0 \), \( m = 1, 2, 3 \), meaning that intergrain current is always smaller than intragrain currents. By applying GA with the previous model to the sample with
Values are shown in Table I and comparisons considered for this example are based on the superposition of intragrain ($J_{C1}$ to $J_{C3}$) and intergrain ($J_{C4}$ and $J_{C5}$) current loops.

III. EVALUATION OF THE PROPOSED METHODOLOGY

In order to validate the modeling approach, trapped flux profiles at different heights (2, 4 and 10 mm) were analyzed for sample #2. Flux density contours at the different heights are represented in Fig. 7. Field profiles at $y = 0$ mm are compared in Fig. 8.

A. Current densities determination using different height measurements

Using the proposed methodology, current densities were determined for each of the heights, as well as their averages. Numerical values are shown in Table I and comparisons between these are represented in Fig. 9. Ideally, each $J_{Cm}$, $m = 1 \ldots 5$ would be independent of the measurement height used for its determination.

![Sand pile modeling extension of three seeds samples consisting on the superposition of intragrain ($J_{C1}$ to $J_{C3}$) and intergrain ($J_{C4}$ and $J_{C5}$) current loops.](image1.png)

![Normal component of flux density of an artificial three seeds sample determined by extended sand pile modeling.](image2.png)

**TABLE I**

<table>
<thead>
<tr>
<th>Current density (A/mm²)</th>
<th>$Z_0 = 2$ mm</th>
<th>$Z_0 = 4$ mm</th>
<th>$Z_0 = 10$ mm</th>
<th>Average value (A/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{C1}$</td>
<td>103.30</td>
<td>71.40</td>
<td>69.95</td>
<td>81.55</td>
</tr>
<tr>
<td>$J_{C2}$</td>
<td>80.82</td>
<td>58.85</td>
<td>50.54</td>
<td>63.40</td>
</tr>
<tr>
<td>$J_{C3}$</td>
<td>89.01</td>
<td>78.68</td>
<td>71.00</td>
<td>79.56</td>
</tr>
<tr>
<td>$J_{C4}$</td>
<td>16.87</td>
<td>23.77</td>
<td>15.82</td>
<td>18.82</td>
</tr>
<tr>
<td>$J_{C5}$</td>
<td>31.96</td>
<td>25.88</td>
<td>34.62</td>
<td>30.82</td>
</tr>
</tbody>
</table>

In order to assess the quality of the modeled flux density distributions with extracted current densities, NRMSD values were calculated for all heights. These are plotted in Fig. 10, where it is clear that $\varepsilon$ is much higher when currents evaluated at $Z_0 = 2$ mm are used (except for $\varepsilon$ calculated for this height). This figure shows that $J_{C}$’s inferred at 2 mm lead to poor results if these are used to predict fields distributions at other heights, when compared with $J_{C}$’s inferred at 4 or 10 mm. On the other hand, if $J_{C}$’s are averaged, this effect is attenuated.

A comparison of real and modeled trapped flux distributions is shown in Fig. 11 for best NRMSD values in each height.

B. Influence of differences of extracted currents on practical applications

In spite of the differences in determined currents at different heights, particularly at $Z_0 = 2$ mm, its influence on predicting flux density distributions at other plans, as required in the methodology described in [4], needs to be performed.

![Contour lines of sample #2 trapped flux, measured at heights 2, 4 and 10 mm.](image3.png)

![Trapped flux density profiles at $y = 0$ mm in sample #2.](image4.png)
Fig. 9. Comparison of values of currents determined by the extended methodology applied at measurements at 2, 4 and 10 mm height. Average value is also shown.

Fig. 10. Comparison of NRMSD, ε, for values of currents determined by the extended methodology applied at measurements at 2, 4 and 10 mm height. NRMSD calculated with average current values is also shown.

Fig. 12. Comparison of NRMSD between average flux profiles calculated with currents evaluated at different heights and average flux profile obtained from real measurements.

IV. CONCLUSIONS

In this paper, a methodology for modeling multiseeded bulk superconducting samples is proposed, as an extension to previous work were some simplifications were made, namely assuming equal grain boundary characteristics. By measuring trapped flux distribution in samples it is possible to use this methodology in designing devices employing these materials. A linear motor described elsewhere is used as application, but other devices are envisaged, as e.g. flywheels with superconducting bearings. Special care has to be taken when using single plan measurements, as is demonstrated when $Z_0 = 2$ mm. Differences observed may be due to incorrect height measurement (owing to materials contraction), but this is masked by averaging current densities evaluated at distinct heights. If currents inferred only at this height were used to derive field values at other height (as in the case of the linear motor previously mentioned), then considerable errors would be expected when deriving e.g. developed forces. On the other hand, using averaged values compensates these errors. One limitation of the current work is related with correct definition of grain sizes. These will be automatically determined in future work, integrating this issue in the optimization problem solved by the genetic algorithms.

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REFERENCES


